FARTHER CONSIDERATIONS

ON THE

COMPARATIVE OBSERVATIONS OF THE DIFFERENCES OF RIGHT ASCENSION OF THE MOON'S ENLIGHTENED LIMB WITH THE SUN'S CENTRE, AND WITH STARS UNDER DIFFERENT MERIDIANS;

TOGETHER WITH THE

ACTUAL RESULTS OF A GREAT NUMBER OF SUCH OBSERVATIONS, MADE IN VARIOUS YEARS, AT THE RESPECTIVE OBSERVATORIES OF GREENWICH AND ARMAGH.

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Observatory, Armagh, May 1, 1806.

HAVING now, for above ten years, been in the practice of either observing myself, or of causing to be observed by my assistant, the moon's meridional passages compared with the most suitable of those fixed stars, whose places are so accurately settled by our excellent astronomer royal, Dr. Maskelyne, and being annually furnished with those accurate and valuable observations, made and published under his inspection, I have been thus provided with complete materials for ascertaining the merit of the method I formerly submitted to the Royal Irish Academy, for determining the longitudes of

places by a comparison of the true differences of the difference of AR. of the ☽'s enlightened limb on the meridian from one or more stars, with which it has been also compared on the same day, under the different meridians, whose longitudinal distances from each other it is required to determine.

I have, in consequence, made a considerable use of these materials; and from a number of careful and repeated investigations, I am enabled to decide that this method is capable of the highest degree of accuracy; is easily put into practice by persons who are furnished with only a good time-keeper, and a portable transit instrument of no very high power or great size; and which, with a simple apparatus, may be set up in a few hours, under even a bell tent or other slight observatory: and from these considerations it appears, that this method is peculiarly adapted to the use of scientific persons, who may chance to be employed on voyages, either of commerce or discovery. When I formerly presented to the Academy a paper on this subject, I was not so well aware of *the practical facility of this method, or of the degree of accuracy, of which it was capable in practice; I spoke and wrote on it, of course, with more diffidence, and as rather suited to the communications of astronomers, in the neighbouring

* I have been strongly confirmed in my opinion of its *practical facility* by the successful application of a small transit instrument of about six inches focal distance, an achrometic object glass, twenty times mag. power, and system of three par. wires, used occasionally at my glebe residence under a meridian 30ˢ of time E. of the observatory of Armagh.

bouring observatories of Europe, than to distant situations and less expert hands.

I am now convinced that, by attending to the *method of observation, suggested by Dr. Maskelyne, and with reasonable practice, any person who has a moderate turn for making astronomical observations, *may*, with a portable transit instrument of three feet length, and from 30 to 50 times magnifying power, furnished with a system of five fine wires, and a clock, whose rate is carefully ascertained, *make* these observations so accurately, that he will soon, from a mean of the five wires, obtain the apparent difference of the AR. of the moon's enlightened limb and a star, true to about one-tenth of a second of time.†

 If

* It may not be useless here, to refer such persons as may be induced to pursue this method of observing the differences of AR. of the heavenly bodies by the transit instrument and time-piece, to the note at the end of the Greenwich observations of passages of stars, for the year 1795, where they will find some very delicate and useful observations on the niceties of astronomical observations by the clock and transit instrument; and, among others, the following precept, on the method of observing, introduced by Dr. Bradley, which Dr. Maskelyne strongly inculcates the necessity of first *duly understanding*, and then, *closely adhering to*. "We should observe with all our attention, when the star comes near the wire, and *fix* (as if we could mark down) the apparent places of the star in the field of the telescope, at the *two beats of the clock*, immediately *preceding and following* the transit of the star across the wire, and *thence* estimate and note down the proper second and tenth answering to the actual transit across the wire. If we are not quick in fixing the place of the star, at the time of the beat, we shall be apt to assign it too backward a place in the telescope, and consequently reckon the time of the transit too great. A good ear seems, in this kind of observation, to be almost as useful as a good eye."

† When the moon has gained 48′ of AR. in time, while the earth has revolved once on its axis, a change of AR. of 2′ of such time indicates that 1ʰ. or 15°. of the

If this method for the longitude is applied to determine the relative positions of meridians, considerably distant from each other, it will be necessary to attend the more particularly to all the corrections of the apparent difference of the differences of AR.; and also to the obtaining of a correct lunar rate for the mid-interval of the observations. To accomplish this part of the process with due precision, it were much to be wished, that the AR.'s of the moon, for noon and midnight, were to be strictly calculated to the nearest second; and in the nautical almanac columns of these AR.'s, given for portions of sydereal time. Till this be done, it will be desirable for those, who wish for particular accuracy in the results of their observations, to turn the ☽'s longitudes, as given to the nearest second, in p. v. of each month in the nautical almanac, into AR.'s; which, though perhaps too tedious and laborious, to be done in general for several observations, might, however, be readily undertaken for one valuable and important one.

As the correct knowledge of the moon's rate at the time of her mid-interval, between the different places of observation is essential, where the estimate longitude is considerably uncertain, then, the operation for getting the rate at the mid-interval, must be repeated with the first obtained longitude, and the process renewed with the more correct rate, thus elicited.

the terrestrial æquator have revolved on the earth's axis. 1°. for each 8 seconds of AR. so gained by the ☽. A degree in 54½. of lat. is = to 32,700 toises nearly; so that in this lat. a possible error of observation of $\frac{1}{10}$ of a second is equal to an error of 400 toises, or less than half an Irish mile in the longitude.

cited. As a correct *horary rate* must be investigated, it may save some trouble, to omit raising this rate to the quantity of AR. corresponding to 12^h, and use the *horary rate obtained* as a member of the final analogy for the longitude. In this case, the tables of logistic logarithms will supersede the use of the table of assistant logarithms given in my first paper on this subject.* It is also proper to observe, that a mode of correcting the *observed* difference of the differences of the AR. of the moon and star under different meridians, has been suggested to me, by my learned friend, the professor of astronomy in Trinity College, Dublin, which is sufficiently accurate and universal; it is, in fact, only a different mode of effecting the purpose of my table 1. in the above-mentioned paper, and either method may be applied. Previous to submitting to the Academy the results of my numerous comparisons of the observed differences of AR. of the ☽. the stars, and the sun, for to obtain the difference of longitude of the observatories

* It will be observed, that Mr. Brinkley applies his correction of the observed difference to the longitude immediately deduced from the entire observed difference, by substracting from the longitude thus obtained, the whole observed difference of the differences in sydereal time, of which he says, " this method " of correction is universal, and may be explained thus:—

" In the time the moon increases her AR. by the observed difference (D) of the differences of AR of ☽ and * each meridian describes an angle in time A = equal to the angular distance of the meridians (L) + D. But, as horary increase of AR. : D. :: 1^h : A′ and A′ being reduced to *sydereal time, gives A: and L = A — D or if L be taken = to A′ — D reduced to sydereal time, it will be sufficiently accurate."

* For the horary increase being calculated to *one hour of solar time*, the result will come out, a portion solar time also, and therefore to be reduced to sydereal time.

observatories of Greenwich and Armagh, I shall give an example at length, of the deduction of the longitude in each method; and also a short table of the actual apparent differences of AR. of the principal fixed stars, observed on the same days at Greenwich and Armagh, that every reader may thus judge of the eligibility of each different method, and, also, clearly see the degree of precision to be expected from such observations, in which various observers, different timepieces, and transit instruments, generally of different powers, are necessarily concerned, when made with due degrees of care and skill in the adjustments, application and use of such instruments.

EXAMPLE.

EXAMPLE.

October 26, 1803.—The observed difference of the differences of AR. of the ☽'s enlightened limb, and α Aquarii at Greenwich and Armagh observatories, was 59″,38. S. T. Correction of observed differences from T. 1. — 2″,10. leaving the corrected difference of differences 57″,28. = to 14′,19″. in measure. The ☽'s change of AR. in 12ʰ. at the rate of the mid-interval of the observations, was 6°. 30′. and her horary rate 32′.30″.

Solution by change of AR. *in* 12ʰ. *and assistant Logarithms.*

Assistant Logarithms of change of ☽, AR. in 12ʰ. -	9.7337.
Prop. Log. of considered difference of differences 14′. 19″.	1.0994.
Prop. Log. of 26′. 26″. long. W. - - - -	0.8331.

Solution by horary rate and proportional Logarithms.

Ar. Comp. Prop. Log. of 32′. 30″. - - - - -	9.2566.
Prop. Log. of 1 hour, - - - - -	0.4771.
Prop. Log. of considered difference of differences	1.0994.
Prop. Log. of 26′. 26″. long. W. - - - -	0.8331.

Solution

Solution by logistic Logarithms, and Mr. Brinkley's correction.

Ar. Comp. of logistic Logarithm of ☽'s horary rate, - -	9.7337.
Logistic Logarithm of the entire observed difference of differences,	0.6065.
	0.3402.
Logistic Logarithm of 27′. 24″,7 approximate longitude.	
59, 38 Mr. Brinkley's correction.	
26′. 25″,32 longitude W.	

A Table

A Table of the differences of the difference of AR. of fixed Stars, as observed, on or about the same days, at the Observatories of Greenwich and Armagh, in the years 1802, 1803, *expressing only the seconds and tenths.*

1802.

	Observed differences at Armagh.	Names of the Stars.	Observed differences at Greenwich.	Difference of differences of AR. Greenwich.
January.				
12,	53″,07.	α Andromedæ. γ Pegasi.	52,8.	— 0″,27.
17,	53,6	α Lyræ. α Aquilæ.	53,66.	+ 0,06.
February				
10,	53,75.	α Lyræ. α Aquilæ.	53,51.	— 0,24.
12,	45,33,	Rigel. β Tauri.	45,6.	+ 0,27.
27,	30,9.	Aldebaran. Capella.	30,75.	— 0,15.
March.				
3, 4,	24,09.	Rigel. Syrius.	24,14.	+ 0,05.
9,	56,49.	Capella. Rigel.	56,68.	+ 0,19.

	Observed differences at Armagh.	Names of the Stars.	Observed differences at Greenwich.	Difference of differences. Greenwich.
March.				
14,	15,45.	Procyon. Pollux.	15,5. 15,4. } 45.	0
April.				
13,	15,24.	Procyon. Pollux.	15,36.	+ 0,12.
13,	38,63.	Pollux. Regulus.	38,50.	— 0,13.
May.				
8,	1,08.	Spica Virgs. Regulus.	1,2	+ 0,12.
15,	51,55.	Spica Virgs. Arcturus.	51,85.	+ 0,3.
17,	50,0.	β Leonis. Spica Virgs.	49,81.	— 0,19.
27,	15,03.	Procyon. Pollux.	15,.24,	+ 0,21.
July.				
13,	19,63.	Antares. α Herculis.	19,7.	+ 0,07.
13,	7,56.	α Herculis. α Ophiuci.	7,42.	— 0,14.
August.				
11,	13,58.	γ Aquilæ. α Capricorni.	13,38.	— 0,20.

	Observed differences at Armagh.	Names of the Stars.	Observed differences at Greenwich.	Difference of differences. Greenwich.
August.				
23,	16,62.	γ Aquilæ. / α Aquilæ.	16,42.	— 0,20.
26,	16,32.	γ Aquilæ. / α Aquilæ.	16,66.	— 0,34.
Septem.				
7.	28,06	α Aquilæ. / β Aquilæ.	7th. 27,9. / 8th. 28,0.	Mean — 0,07.
11.	30,28.	α Aquilæ. / α Aquarii.	30,04. / —,04. f. rate at	— 0,24.
17.	28,94.	β Aquilæ / 2 α Capricorni.	28,98.	+ 0,04.
17.	42,79.	2 α Capricorni. / β Tauri.	42,76.	— 0,03.
28.	44,8.	γ Aquilæ. / β Aquilæ.	44,6.	— 0,2.
28.	13,38.	γ Aquilæ. / 2 α Capricorni.	13,54.	+ 0,16.
October.				
10.	52,71.	α Andromedæ. / γ Pegasi.	52,72.	+ 0,01.
15.	52,88.	β Aquilæ. / α Orionis.	53,08.	+ 0,2.
Decemb.				
1.	30,18.	α Aquilæ. / α Aquarii.	30,08.	— 0,1.

	Observed differences at Armagh.	Names of the Stars.	Observed differences at Greenwich.	Difference of differences. Greenwich.
December				
22.	55,01.	α Lyræ. α Aquilæ.	54,72.	— 0,29.
June omitted.				
10.	51,46.	Spica Virgs. Arcturus.	51,66.	+ 0,2.

Sum of the Greenwich + 2,34. Sum of the Greenwich — 2,49.

1803.

				Greenwich.
January.				
2,	54,42.	α Arielis. α Ceti.	54,44	+ 0,02.
12,	57,20.	α Aquilæ. γ Pegasi.	56,92	— 0,28.
March.				
3.	55,37.	Capella. Rigel.	54,95	— 0,42.
5,	58,08.	Castor. Procyon.	57,84	— 0,24.
3,	46,39.	Rigel. β Tauri.	46,77	+ 0,38.
8,	55,3.	Capella. Rigel.	55,0	— 0,30
9.	46,4.	β Tauri. Rigel.	46,7	+ 0,3.

	Observed differences at Armagh.	Names of the Stars.	Observed differences at Greenwich.	Difference of differences. Greenwich.
March.				
9.	37,43.	β Tauri. Syrius.	37,18.	— 0,25.
29.	16,0.	Procyon. Pollux.	16,08.	+ 0,08.
31.	16,02.			+ 0,06.
April.				
5.	15,78.	Procyon. Pollux.	16,04.	+ 0,26.
May.				
6.	15,92.	Procyon. Pollux.	16,08.	+ 0,16.
June.				
25.	50,9.	Spica Virgs. Arcturus.	50,99.	+ 0,09.
28.	50,9.	Spica Virgs. Arcturus.	51,0.	+ ,10
July.				
10.	56,1.	Arcturus. Aldebaran.	55,8.	— ,30.
August.				
6.	7,54.	α Herculis α Ophiuci.	7,49.	— ,05.

Sept. 5. Mr. Troughton ground down the Pivots of Greenwich T. Inst$^{mt.}$ by Hand.

11.	15,65.	Procyon. Pollux.	16,16.	+ ,51.
27.	28,22.	α Aquilæ. β Aquilæ.	28,0.	— ,22.
October.				
6.	16,66.	Procyon. Pollux.	16,06	— ,6.

Oct. 12. The axes of G. T. Inst^mt. being found imperfect were ground true in a Lathe.

October.	Observed differences at Armagh.	Names of the Stars.	Observed differences at Greenwich.	Difference of differences. Greenwich.
22.	16,58.	γ Aquilæ. / α Aquilæ.	16,6.	+ ,02.
22.	27,94.	α Aquilæ. / β Aquilæ.	28,0.	+ ,06.
22.	13,88.	γ Aquilæ. / 2 α Capricorni.	14,04.	— ,16.
22.	57,30.	α Aquilæ. / 2 α Capricorni.	57,44.	+ ,14.
26.	29,23.	Fomalhaut. / α Andromedæ.	29,38.	+ ,15.
Nov. 4.	44,64.	γ Aquilæ. / β Aquilæ.	44,5.	—;14.
21.	12,41.	Fomalhaut. / α Pegasi.	12,58.	+ ,17.
21.	5,19.	α Aquarii. / Fomalhault.	5,22.	+ ,03.
23.	42,7.	α Andromedæ. / γ Pegasi.	42,6.	— ,10.
Dec. 4.	50,34.	Spica Virgs. / Arcturus.	50,34.	0
27.	52,7.	Capella. / Rigel.	52,53.	— ,17.

NOTE.—It will be observed, that since the last repair of the axis of the Greenwich T. inst. the difference of the observed differences of AR. of any two stars never amounts to two-tenths of a second, so that if one-tenth of a second be allowed as the probable error of observation and counted as will appear from the sum of the + and — differences in the opposite directions, the observed places of the several stars will be found to agree to half-a-tenth of a second in each place. The sum of the + differences at Greenwich is, 57, and of the —, 57. The sum of the + differences in 1802 is 2,34, and of the — 2,49.

A view

A view of the mean and extreme Results, of the difference of differences of AR. of the ☽ and the same fixed Stars, observed on the same days at the observatories of Greenwich and Armagh, to each of which if 4″,2 are added, the sum will be the difference of longitude of the two observatories; taken in the year 1795.

Days.	Results.	Days.	Results.	Days.	Results of Ex. Observations.	Mean of Extreme Observations in their results.
Jan.		Oct.		Jan.		
3.	26′.25″	25.	26ˢ.21ᵐ	29.	26ˢ.15,ᵐ3	26′.25ˢ,7
Feb.						+ 4,2
14.	26ˢ22ᵐ	26.	26ˢ.28ᵐ	28.	26′.16″	26′.29″,9. Lon. West.
March.				March		
30.	26ˢ.27ᵐ,5	29.	26ˢ.26ᵐ,3	1.	26′.13″	
		Nov.				* *M. Results* very less than 10″ from the Mean.
31.	26ˢ.22″,3	24.	26ˢ.26ᵐ,3	28.	26ˢ.42ᵐ	
July.		Dec.		July.		* *Ex. Results* very more than 10″ from the Mean, or about 10ᵍ
1.	26ˢ.29ᵐ	29.	26ˢ.30ᵐ	25.	26′.35″	
Aug.			End of Mean Results	Oct.		
26.	26ˢ.26ᵐ			5.	26′.18″,3	
27.	26ˢ.27ᵐ		Mean of these. 26ˢ.25ᵐ88 + 4,35	27	26′.35″	
Sep.			26ˢ.30″,23 Lon. West.	Dec.		
28.	26ˢ.26ᵐ			2.	26′.31″	

Observations for the year 1797.

July	1 Limbs.	July.	2 Limbs.
3.	26.″14,7	17.	32″,3
Sep.		Nov.	
28.	26.17,7	10.	37″,7
Dec.		Dec.	
2.	26.20,0	12.	32″3

Mean Result of these 26′.25″,78
To make the Result Syd. T. + 4,35
Lon. West. 26. 30, 13

The reductions of these observations were not reassumed till those of the years 1802 *and* 1803, *which were done in the present year* 1806, *and give the following results:*

1802. Jan.	1 Limbs.	1802. May.	2 Limbs.	Mean, 1 Limbs in 1802.	
11.	26.20	17.	26.53	26.27.	26.26.
14.	26.13	25.	26.41	26.29½	26.28.
Feb.		Aug.			
12.	26.15	11.	26.36⅓	26.30.	26.33.
		Sept.			
15.	26.20.	17.	26.34¼	26.29.	26.21.

1802.	1 Limbs.	1802.	2 Limbs.	Mean, 1 Limbs in 1802.	
March.		Sep.			
13.	26.20.	17.	26.34.¼.	26.33½.	26.23.
May.					
6.	26.16.	17.	26.35.0.	23.29.	26.36.
		Oct.			
7.	26.20.	9.	26.19½.	26.32½.	
Nov.					
8.	26.19⅓.	15.	26.22.	26.23½.	
8.	26.14⅓.	19.	26.30.	26.25⅓.	
Dec.					
5.	26.19.		56.24½.		

The mean Result of these 20 Observations is, 26′.25″,32
+ 4, 35
Lon. West. 26′.29″,67

The whole number of comparisons this year 35. The mean Result of the whole of which is 26′.25″,33
+ 4, 35
Lon. West. 26′.29″,68

The whole number of comparisons reduced in the year 1803 was 32. The mean Result of which is 26′.26″,3
+ 4,35
26.30.65

The foregoing results were all obtained from comparisons of the difference of differences of AR. of the ☽ enlightened limb and a star. The following are the results of all the similar observations, obtained of the difference of the differences of AR. of the ☽'s enlightened limb from the ☉'s centre, reduced from the mean of its passages over the 5 wires of the T. Insts. of Armagh and Greenwich, in the year 1803.

Difference of ☉'s and ☽'s enlightened Limb.

Feb.	4.	26\.20″,5.	The mean result of all these differences of ☉, and ☽'s enlightened limb.		26′.26′,,6. + 4. 35.
March	30.	26′,23\\,0.		Lon. West.	26′.30″95.
June	28.	26′.23,\\0.	M. results from inc. for syd. time.	1795.	26′ 30″,23.
July	11.	26,′36″,0.		1797.	26′.30′,13.
Oct.	22.	26\.35″,0.	20 observations in	1802.	26′.29″,67.
	28.	26′.23″,0.	The whole in	1802.	26′.29″,68.
			The whole in	1803.	26\.30″,65.

The correct Lon. W. deduced from a mean of all, is 26′.30″,22.

In the year 1804, *there were but* 18 *good sets of comparative observations of the difference of differences, of* ☽ *and stars AR's, on the meridians of the observatories of Greenwich and Armagh, which being reduced, gave the following results for the Longitude.*

26′,28″,1		
15, 0		
32, 7		
27, 0		
33, 2		
40, 7		
29, 8		
21, 0		
30, 5		
27, 2		
9, 6		
40, 9		
35, 1		
33, 8		
35, 1		
22, 6		
23, 0		
9, 3		
Seconds 494, 6	Lon. from a mean of all these,	26′.27″,45
	For syd. time.	+ 4. 35
		26. 31, 80

If the four extreme observations should be rejected, the Lon. W. will be 26′.31″,05.